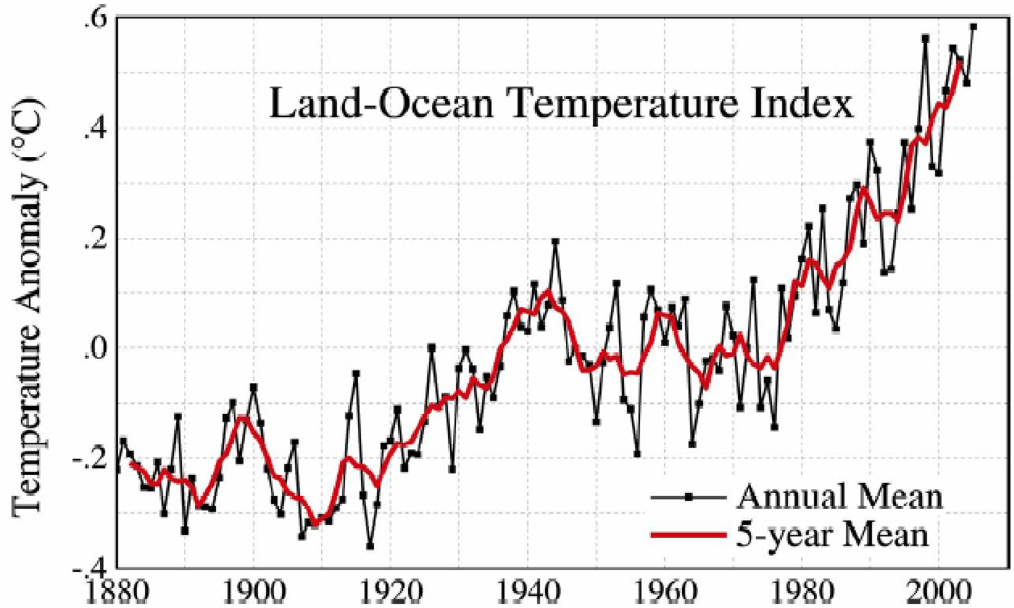

Global mean surface temperature change based on surface air measurements over land and SSTs over ocean

Source: Update of Hansen et al., *JGR,* **106**, 23947, 2001; Reynolds and Smith, *J. Climate,* **7**, 1994; Rayner et al., *JGR,* **108**, 2003.

Increase in Hurricane Intensity with the Rise of Global Temperature: An Analytical Derivation

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Assumntion : Tropical Cyclone is assumed to be a cylindricrd vorlex.

1. Derivation of Maximum velocity

The vcrtical component of Vorticity $\underline{\zeta} = curl \quad \underline{v}$

Assumption vorticity $\underline{\zeta} \qquad for\, r \quad \leq a$
Where a is the point where max velocity occurs
In terms of stream function ψ, the components of velocity $u \,\&\, v$ are given by (in rectangular coordinate system) $u = -\frac{\partial \psi}{\partial y} \qquad v = \frac{\partial \psi}{\partial x}$

$$\text{Thus} \quad \zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \nabla^2 \psi$$

In polar coordinates $v_r = -\frac{1}{r}\frac{\partial \psi}{\partial \theta} \qquad v_\theta = \frac{\partial \psi}{\partial r}$

$$\zeta = \nabla^2 \psi = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2}$$

Assuming symmetry $\qquad \frac{\partial \psi}{\partial \theta} = 0$

Hence $\qquad \zeta = \frac{d^2 \psi}{dr^2} + \frac{1}{r}\frac{d\psi}{dr} = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi}{dr} \right)$

Solution for $\quad r \leq a$

$$\zeta = \frac{1}{r}\frac{d}{dr}\left(r\frac{d\psi}{dr} \right)$$

$$\text{or} \qquad \frac{d}{dr}\left(r\frac{d\psi}{dr} \right) = r\zeta$$

Integrating w. r .to r, $\quad r\frac{d\psi}{dr} = \frac{\zeta r^2}{2} + \mathrm{B}$
where B is a constant

$$d\psi = \frac{\zeta r}{2} dr + \frac{B}{r} dr$$

Further integration gives

$$\psi = \frac{\zeta r^2}{4} + B \log r + D$$

where D is another constant

Since ψ is finite at the origin $B = 0$

$$v_\theta = \frac{d\psi}{dr} = \frac{\zeta r}{2}$$

$$\text{At} \quad r = a, \quad (v_\theta) = \frac{\zeta a}{2}$$

Thus the maximum value of v_θ *is* $\frac{\zeta a}{2}$

The observed Profile tropical cyclone velocity as obtained from aircraft reconnaissance is shown in Fig 1 (Ref. 1).

Fig 1

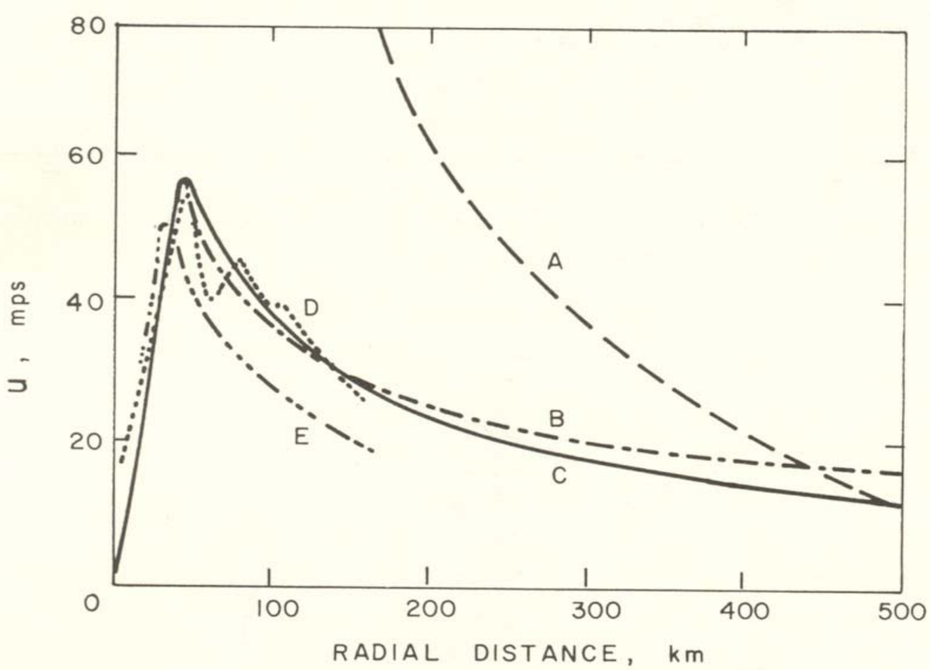

Fig. 92. Increase in rotational (tangential) wind speed, u, approaching a hurricane center. *Curve C* is from J. Malkus and H. Riehl, "On the Dynamics and Energy Transformation in Steady-State Hurricanes"; *B*, a modified Rankine vortex with x = 0.5 (see Eq. 9.3); *A*, a profile with constant absolute angular momentum; *D*, profile observed in hurricane Beulah, 1963, at 2,100-m altitude; *E*, in hurricane Hilda, 1964, at 1,000 m.

We apply the barotropic vorticity equation of Dynamical Meteorology

$\frac{\zeta + f}{H}$ is constant where ζ is vorticity, f is the Coriolies parameter and H is the fluid depth (Ref: 2, 3).

Putting H = $z_2 - z_1$

$\frac{\zeta + f}{z_2 - z_1}$ is constant = k (say)

We apply hydrostatics.

$$dp = -\rho g dz$$

$$= -\frac{pmg}{RT} dz \quad \text{where } T \text{ is the virtual temperate}$$

or $\frac{dp}{p} = -\frac{mg}{RT} dz$

or Integrating $\ell n\, p_1 - \ell n \quad p_2 = \frac{mg}{RT}(z_{2-}z_1)$ ....(5)

Putting (5) in (4) where p_1 and p_2 are pressures and heights respectively at levels 1 and 2.

$$\zeta + f = k(z_2 - z_1)$$

$$= \frac{RT}{mg} k(\ell n\, p_1 - \ell n \quad p_2)$$

In the case of a cyclone $\zeta \quad >> f$ (Ref. 4)

$$v_\theta = \frac{\zeta a}{2} = \frac{RT}{2mg} k(\ell n\, p_1 - \ell n \quad p_2)$$

Which shows that cyclone velocity is directly related to the ambient temperature of the atmosphere.

References:

1. Simpon, Robert H: The Hurricane and its Impact, Page 196, Lousicana State University Press, 1981.

2. Horace Robert Byers: General Meteorology, Page 257, Mc Graw-Hill Book Co. 1959.

3. A. Wiin-Nielsen (Editor), Compendium of Meteorology Vol 1, Part- 1, Dynamic Meteorology Page 313, World Meteorological Organisation, 1981.

4. Richard A. Anthes, Tropical Cyclones, Their Evolution, Structure and Effects, American Meteorological Society, 1982

This approach has been successful in explaining the maximum velocity in tropical cyclones from pressure drop (Ref. 3).